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A ^{137}Cs erosion model with moving boundary

Chuan Yin [a], Hongbing Ji [a, b, *]

[a] Beijing Municipal Key Laboratory of Resource Environment and GIS, College of Resource Environment and Tourism, Capital Normal University, Beijing 100048, China

[b] State Key Laboratory of Environmental Geochemistry, Institute of Geochemistry, Chinese Academy of Sciences, Guiyang 550002, China

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ABSTRACT

A novel quantitative model of the relationship between diffused concentration changes and erosion rates using assessment of soil losses was developed. It derived from the analysis of surface soil ^{137}Cs flux variation under persistent erosion effect and based on the principle of geochemistry kinetics moving boundary. The new moving boundary model improves the basic simplified transport model (Zhang et al., 2008), and mainly applies to uniform rainfall areas which show a long-time soil erosion. The simulation results for this kind of erosion show under a long-time soil erosion, the influence of ^{137}Cs concentration will decrease exponentially with increasing depth. Using the new model fit to the measured ^{137}Cs depth distribution data in Zunyi site, Guizhou Province, China which has typical uniform rainfall provided a good fit with $R^2 = 0.92$. To compare the soil erosion rates calculated by the simple transport model and the new model, we take the Kaixian reference profile as example. The soil losses estimated by the previous simplified transport model are greater than those estimated by the new moving boundary model, which is consistent with our expectations.

1. Introduction

The radioactive caesium isotope ^{137}Cs adheres to clay particles in the topsoil and is lost by particle transport under the action of water erosion. This suggests that short and medium term erosion assessments can be determined as a function of the amount of ^{137}Cs lost. The redistribution of ^{137}Cs after it incorporates into surface soil is a complex process which includes physical, chemical, and biological processes (Zapata, 2002; Walling and He, 1992, 1993; Pegoyev and Fridman, 1978). The factors affecting ^{137}Cs redistribution include soil properties, topography, and land utilization (Zhang et al., 1994; Ritchie and McHenry, 1978; Queralt et al., 2000; McHenry and Ritchie, 1977; Govers et al., 1999; Govers, 1985). ^{137}Cs redistribution models, such as the convection–dispersion model, are usually made using the assumption of no erosion effects (Konshin, 1992; Schuller et al., 1997; Van Genuchten and Cleary, 1979; Walling and He, 1997). This assumption also applies to the compartment model (Boone et al., 1985; Bunzl et al., 1994; Strebl et al., 1996), and other models (Kirchner, 1998). Undisturbed land erosion models include the empirical model (Loughran and Campbell, 1995), the profile-shape model (Zhang et al., 1990), and the diffusion model (Walling and He, 1992; Zhang et al., 2008). The profile-shape model and diffusion models are based on the ^{137}Cs redistribution principle in order to establish the soil erosion model, while the diffusion model is more comprehensive. Diffusion models include the transport model (Walling and He, 1999) and the simplified transport model (Zhang et al., 2008). Although the transport model considers multiple links of ^{137}Cs redistribution, the model structure is complex and has many parameters that are difficult to apply in practice. The simplified transport model is based on the transport model but ignores the convective term of the diffusion equation, simplifies the calculation process, and improves the model feasibility.

Simulation of the ^{137}Cs diffusion process under natural erosion conditions is complex. There are at least two major difficulties: (1) The diffusion boundary (topsoil) will move downward with erosion; (2) While the boundary is eroding, ^{137}Cs will run off from the outside of the system and affect diffusion inside the system. The simplified transport model assumes that erosion occurs only during a short period at the end of each year. It ignores the effect of the erosion process on the ^{137}Cs diffusion under the topsoil. The model is particularly suitable for areas with low annual rainfall and concentrated rainfall, but for areas with a long rainy season exceeding 5 months and average monthly rainfall, the model has

* Corresponding author. State Key Laboratory of Environmental Geochemistry, Institute of Geochemistry, Chinese Academy of Sciences, Guiyang 550002, China.
E-mail address: hongbing_ji@163.com (H. Ji).

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limitations. This paper demonstrates how the model can be improved, on the basis of geochemistry dynamics, for the moving boundary problem.

2. Background

For the ^{137}Cs diffused model on the basis of physical action, the first application to assess soil erosion rate is the one-dimensional diffusion convection model developed by Walling and He, 1999. The model performs simulation for a no erosion soil profile and the ^{137}Cs diffused distribution can be represented by the following equation:

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial}{\partial x}\left(\frac{\partial C(x,t)}{\partial x}\right) - V\frac{\partial C(x,t)}{\partial x} - \lambda C(x,t) \tag{1}$$

where $C(x,t)$ is the ^{137}Cs concentration (mBq cm^{-3}), x is the depth (cm), t is the time (yr), D is the diffusion coefficient (cm^{-2} yr^{-1}), V is the migration rate (cm^{-1} yr^{-1}), and λ is the ^{137}Cs decay rate (0.023 yr^{-1}). Eq. (2) (Walling and He, 1997) is the analytical solution to Eq. (1).

$$\begin{aligned} C(x,t) = & \int_{t0}^{t} e^{-\lambda(t-t')}dt' \int_{0}^{\infty} \frac{I(t')}{h0} e^{-y/H0} e^{\nu(x-y)/2D-\nu^2(t-t')/4D} dy \\ & \times \left\{ \frac{1}{\sqrt{4\pi D(t-t')}} \left[e^{-(x+y)^2/4D(t-t')} + e^{(x-y)^2/4D(t-t')} \right] \right. \\ & \left. - \frac{\nu}{2D} e^{\nu z/D} erfc\left[\frac{x+y+\nu(t-t')}{\sqrt{4D(t-t')}} \right] \right\} \end{aligned} \tag{2}$$

where $I(t')$ represents ^{137}Cs fallout in time t, and $H0$ represents the relaxation depth of the initial distribution of fallout ^{137}Cs in the soil profile (cm), and erfc is the error function. For the erosion profile, the model considers that the undisturbed soil profile has a relatively low erosion rate and the surface position (approximately 0.5 cm) of the ^{137}Cs concentration $C(t)$ may be approximated by use of the non-eroding surface soil represented by the following equation:

$$C(t) \approx C(0,t) \tag{3}$$

According to variation of the surface soil concentration, the erosion rate can be calculated by the following equation (Walling and He, 1999):

$$\int_{0}^{t} PRC(t')e^{-\lambda(t-t')}dt' = Ais(t) \tag{4}$$

where $C(t')$ is the ^{137}Cs concentration in the surface soil of eroded soil (mBq cm^{-3}), $Ais(t)$ is the total loss of ^{137}Cs inventory (mBq cm^{-2}), P is the particle size correction factor, and R is the erosion rate (cm yr^{-1}).

The transport model incorporates many factors, which brings it relatively close to the actual diffusion process, but there are three main issues: Firstly, to use the model one needs to know the fallout fluxes of ^{137}Cs for each year, which in different parts of the world, varies considerably and accurate data are difficult to obtain. Secondly, the model contains convective terms. When the boundary conditions are complex, the data value resolution can only be obtained by the finite difference method and it is difficult to obtain analytical solutions. Thirdly, when the erosion rate is large, the erosion profile surface ^{137}Cs concentration is not equal to the no erosion soil surface concentration. In order to solve the above problems, the simplified transport model has two assumptions about the no erosion soil profile redistribution of ^{137}Cs: First, the year-by-year ^{137}Cs settling volume was completed in 1963. Namely the settling volume in 1963 as equal to the total settling volume of all years, by which assumptions about the diffusion type can be seen as an instant surface source diffusion. Second, many studies have indicated that the convection effect on the ^{137}Cs migration is very small, almost zero, so the convective terms can usually be removed from the model (Bossew and Strebl, 2001; Schuller et al., 1997; Shinonaga et al., 2005; Szerbin et al., 1999). The following equation is the simplified model formula:

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial}{\partial x}\left(\frac{\partial C(x,t)}{\partial x}\right) - \lambda C(x,t) \tag{5}$$

The solution of Eq. (5) is Eq. (6), the solution of which is identical in form to the Gaussian distribution. Such problems can be generally resolved by statistical methods (Zhang et al., 2008; Zhang, 2010).

$$C(x,t) = Me^{-\lambda t}\frac{e^{-\frac{1x^2}{4Dt}}}{\sqrt{D\pi t}} \tag{6}$$

where M is the initial ^{137}Cs inventory in 1963 (mBq cm^{-2}).

Eq. (6) is the ^{137}C diffusion model of non-eroding soil profiles, for which the numerical model is shown in Fig. 1(a). For the soil erosion profile, the simplified transport model makes the assumption that the soil erosion of each year only occurs within a short interval during the rainy season, so erosion can be put into the last period of each year, as shown in Fig. 1(b). The erosion rate computational formula is shown (Zhang et al., 2008) as

$$\begin{aligned} Arm(T) &= \int_{H}^{+\infty} \frac{Arm(T-1)e^{-\lambda}}{\sqrt{\pi DT}} e^{-x^2/4DT} dx \\ &= Arm(T-1) erfc\left[\frac{H}{2\sqrt{DT}}\right] \end{aligned} \tag{7}$$

where $Arm(T)$ is the remaining ^{137}Cs inventory in the profile during the sampling year (mBq cm^{-2}), T is the time elapsed from 1963 (yr), and H is the annual soil loss depth during the period from 1963 to the sampling year (cm).

Eq. (7) can be rewritten as

$$Arm(T) = Aref(T)\prod_{i=1}^{T} erfc\left[\frac{H}{2\sqrt{Di}}\right] \tag{8}$$

In the year T, $Aref(T)$ is the ^{137}Cs inventory in the reference profile (mBq cm^{-2}). By Eq. (8), the annual soil erosion can be assessed by comparing the ^{137}Cs inventory in the reference profile.

The simplified transport model solves the above-mentioned two problems, but there are other limitations including the assumption regarding the erosion process. The model assumes that erosion occurs during the last period at the end of each year and ignores the effect of the erosion process on the erosion of ^{137}Cs diffusion under the topsoil. This assumption is suitable in arid land areas, such as Northwest China. These regions have little annual rainfall and it occurs mainly focused during the rainy season, which is about one or two months long. For the regions that are easily affected by the monsoon, this assumption no longer corresponds with the actual situation. Examples include most areas of southern China which have relatively more rainfall. In some areas, there is also a considerable amount of rainfall in the winter. These areas must consider erosion impacts on the diffusion process. When ignoring this factor, the $A_{rm(T)}$ in Eq. (7) will be greater than the actual value and the erosion rate H in Eq. (8) will be greater than the actual value. This paper directly analyzes erosion profiles of soil surface ^{137}Cs flux

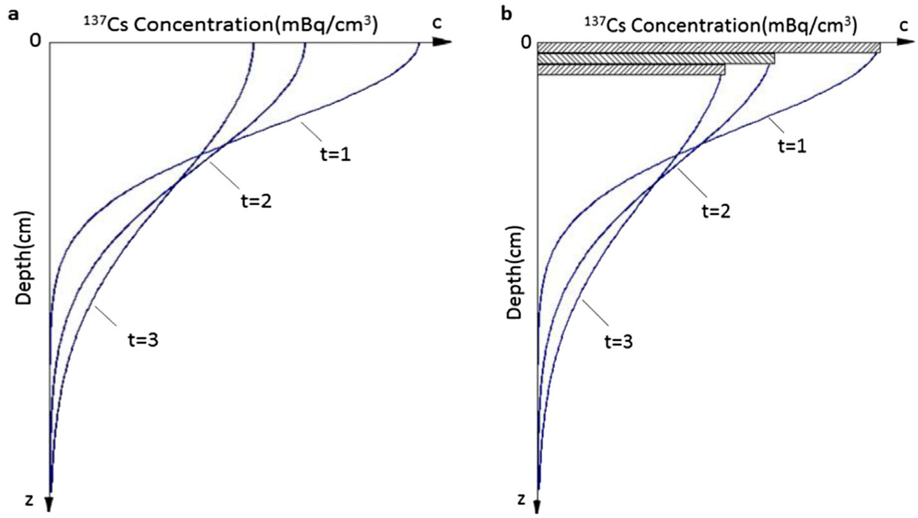

Fig. 1. ^{137}Cs transport process in undisturbed soil profile.

variation and builds the soil erosion model based on the moving-boundary to address the above problems.

3. Establishment of the moving-boundary model

The moving-boundary model uses two assumptions from the simplified transport model: it does not consider convection effects, and the initial condition is an instantaneous surface source. At the initial time (1963) the ^{137}Cs is set at $x = 0$ with a very high concentration of the total mass M, for which the concentration is seen as infinite. The ^{137}Cs diffusion equation for a no erosion profile that is associated with the laboratory reference frame can be described as

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial}{\partial x}\left(\frac{\partial C(x,t)}{\partial x}\right) - \lambda C(x,t) \tag{9}$$

The ^{137}Cs diffusion in erosion profiles considers the soil layer as a semi-infinite medium, as shown in Fig. 2.

In Fig. 2, at $t = 0$, the interface between the soil and air is $x = 0$, and the surface layer suffering erosion effects at time t moves to x_0, which is a moving depth boundary. The movement speed (erosion rate) is u, and we assume that movement speed is a constant. During the erosion process, the boundary moves downwards. The relationship between the interface position and time is

$$x0 = ut \tag{10}$$

where $x0$ is the erosion depth (cm), u is the erosion rate (cm yr^{-1}), t is the time (yr).

To better evaluate the material migration at the boundary we converted the coordinate system, so the laboratory fixed interface frame was converted to an interface fixed reference frame. Under this new reference frame, the coordinates are expressed as y, and the two-phase (air and soil) interface is fixed at $y = 0$. The specific convention is

$$y = x - x0 \tag{11}$$

$$tnew = t \tag{12}$$

To integrate Eq. (11) and Eq. (12) into Eq. (9), the new diffusion equation having been transformed to the new reference frame is as follows:

$$\frac{\partial C}{\partial t} = D\frac{\partial}{\partial y}\left(\frac{\partial C}{\partial y}\right) + u\frac{\partial C}{\partial y} - \lambda C \tag{13}$$

Eq. (13) is similar to Eq. (1), but they have a completely different physical significance. V is a convective term (cm^{-1} yr^{-1}) and u is the erosion rate (cm yr^{-1}), and the coordinate system of the two equations is different.

After the coordinate system change, the initial condition of the differential equation, time $t = 0$, remains as

$$C(y,0) = M\delta(y) \tag{14}$$

where M is the initial ^{137}Cs total amount in 1963 (mBq cm^{-2}), δ is impulse function.

The species flux at the interface $y = 0$ is

$$J = -D\frac{\partial C}{\partial y}\bigg|_{y=0} \tag{15}$$

where J is the ^{137}Cs flux in the soil surface (mBq cm^{-3} yr^{-1}).

^{137}Cs, at the boundary, that is adhering to clay particles and is being lost as sediment with rainwater can be seen as the same diffusion problem as that of heat conduction for a one-dimensional

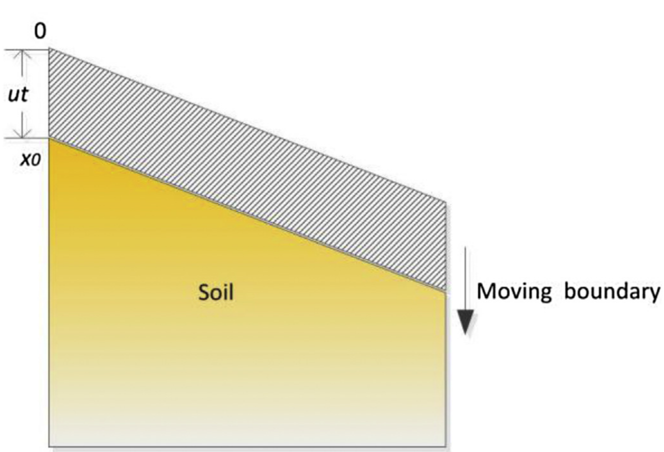

Fig. 2. Erosion soil profile of ^{137}Cs diffusion.

bar in contact with a mobile fluid (e.g., air) at the boundary. In this case, the first boundary condition or the second boundary condition is improper, so it is necessary to use the third boundary condition (Newton's cooling theorem), namely at the boundary, for which the heat flux of the outflow pole that is approximate by the ground and bar is proportional to the external temperature difference. At the left end point (y = 0), the ^{137}Cs loss group component is

$$-u[C(0,t) - CB(t)] \tag{16}$$

$CB(t)$ represents the ^{137}Cs concentration of the external air, and here is $CB(t) = 0$. Here, it should be noted that $u > 0$ and in front of the symbol u is a negative, and diffusion in this paper is a semi-infinite media without the right boundary, only with the left boundary. If it is diffusion in a finite media, the first symbol u at the right boundary is positive. The ^{137}Cs loss group component Eq. (15) should be equal to the diffusion fluxes Eq. (16), which is

$$D\frac{\partial C}{\partial y} - uC = 0\Big|_{y=0} \tag{17}$$

where D is the diffusion coefficient (cm^{-2} yr^{-1}), y is the depth (cm), and u is the erosion rate (cm yr^{-1}).

4. Model solving method

In Eq. (13), λC is the concentration change caused by decay, and due to the radioactive component diffusion equation it is necessary to multiply both sides of the equation at the same time by $e^{\lambda t}$ to simplify the model and to eliminate the decay term, thus Eq. (13) is converted to

$$\frac{\partial \omega}{\partial t} = D\frac{\partial}{\partial y}\left(\frac{\partial \omega}{\partial y}\right) + u\frac{\partial \omega}{\partial y} \tag{18}$$

where $\omega = Ce^{\lambda t}$.

In summary, the soil erosion profile for the ^{137}Cs diffusion model is as follows:

$$\frac{\partial \omega}{\partial t} = D\frac{\partial}{\partial y}\left(\frac{\partial \omega}{\partial y}\right) + u\frac{\partial \omega}{\partial y}(0 < y < +\infty, t < +\infty) \tag{19}$$

$$\omega(y,0) = M\cdot\delta(y-0) \tag{20}$$

$$D\frac{\partial \omega}{\partial y} - u\cdot\omega = 0\Big|_{y=0} \tag{21}$$

where Eq. (20) is the initial condition for Eq. (19), and Eq. (21) is the boundary condition for Eq. (19). In order to solve Equations 19–21, it is necessary to eliminate the first order term $u\frac{\partial\omega}{\partial y}$ in Eq. (19), therefore, we let

$$\omega(y,t) = e^{ay+bt}\cdot K(y,t) \tag{22}$$

where a and b are undetermined constants, and $K(y,t)$ is intermediate variable.

Calculating the derivative of Eq. (22) with the variable t and y as follows:

$$\frac{\partial \omega}{\partial t} = \left[\frac{\partial K}{\partial t} + bK\right]e^{ay+bt} \tag{23}$$

$$\frac{\partial \omega}{\partial y} = \left[\frac{\partial K}{\partial y} + aK\right]e^{ay+bt} \tag{24}$$

$$\frac{\partial^2 \omega}{\partial t^2} = \left[\frac{\partial^2 K}{\partial t^2} + 2a\frac{\partial K}{\partial y} + a^2K\right]e^{ay+bt} \tag{25}$$

By bringing Equations 23–25 into Eq. (19), then

$$\frac{\partial K}{\partial t} + bK = D\left(\frac{\partial^2 K}{\partial t^2} + 2a\frac{\partial K}{\partial y} + a^2K\right) + u\left(\frac{\partial K}{\partial y} + aK\right) \tag{26}$$

So Eq. (26) can be rewritten as

$$\frac{\partial K}{\partial t} = D\frac{\partial^2 K}{\partial y^2} + (2aD+u)\frac{\partial K}{\partial y} + \left(a^2D + au - b\right)K \tag{27}$$

Eliminating the first order term $\frac{\partial K}{\partial y}$ and the constant term K, Let

$$2aD + u = 0, a^2D + au - b = 0 \tag{28}$$

Therefore, we have:

$$a = -\frac{u}{2D}, b = -\frac{u^2}{4D} \tag{29}$$

Hence, it is clear that

$$\omega(y,t) = e^{-\frac{uy}{2D}-\frac{u^2t}{4D}}\cdot K(y,t) \tag{30}$$

From the above formula, it is implied that $K(y,t)$ should satisfy the equation

$$\frac{\partial K}{\partial t} = D\frac{\partial^2 K}{\partial y^2} t > 0, y > 0 \tag{31}$$

And its definite condition satisfying

$$K(y,t)\Big|t = 0 = M\cdot\delta(y)e^{-\frac{uy}{2D}} \tag{32}$$

$$\left[\frac{\partial K}{\partial y} - \left[\frac{u}{D} + \frac{u}{2D}\right]K\right]y = 0 = 0 \tag{33}$$

where Eq. (32) is the initial condition for Eq. (19), and Eq. (33) is the boundary condition for Eq. (31).

We have the following mathematical equation:

$$(\mathrm{I})\begin{cases} \dfrac{\partial s}{\partial t} = a^2\dfrac{\partial^2 s}{\partial z^2} \\ s(t,z)|t = 0 = f(z) \\ \dfrac{\partial s}{\partial z} - \beta s\Big|z = 0 = 0 \end{cases}$$

And we know its solution is

$$s(t,z) = \frac{1}{2a\sqrt{\pi t}}\int_0^\infty \left\{f(\xi)\left[e^{\frac{(z-\xi)^2}{-4a^2t}} + e^{\frac{(z+\xi)^2}{-4a^2t}}\right] - 2\beta e^{\frac{(z+\xi)^2}{-4a^2t}-\beta\xi}\int_0^\xi e^{\beta\eta}f(\eta)d\eta\right\}d\xi \tag{34}$$

Thus, we have

$$K(t,z) = \frac{1}{2\sqrt{D\pi t}}\int_0^\infty \left\{M\cdot\delta(\xi)\cdot e^{-\frac{u\cdot\xi}{2D}}\left[e^{\frac{(z-\xi)^2}{-4a^2t}} + e^{\frac{(z+\xi)^2}{-4a^2t}}\right] - 2\beta e^{\frac{(y+\xi)^2}{-4Dt}-\beta\xi}\int_0^\xi M\cdot\delta(\eta)\cdot e^{-\frac{u\cdot\eta}{2D}}e^{\beta\eta}d\eta\right\}d\xi \tag{35}$$

where $\beta = \frac{3u}{2D}$.

Now by virtue of

$$\int_0^{\infty}\{M\cdot\delta(\xi)\cdot e^{-\frac{u\cdot\xi}{2D}}\left[e^{\frac{(y-\xi)^2}{-4a^2t}}+e^{\frac{(y+\xi)^2}{-4a^2t}}\right]d\xi = Me^{-\frac{y^2}{4Dt}} \tag{36}$$

$$\int_0^{+\infty}2\beta e^{\frac{(y+\xi)^2}{-4a^2t}-\beta\xi}\int_0^{\xi}M\cdot\delta(\eta)\cdot e^{-\frac{u\cdot\eta}{2D}}e^{\beta\eta}d\eta d\xi$$
$$= e^{\beta(D\beta t+y)}\sqrt{\pi}\beta\left\{1-erf\left\{\frac{2D\beta t+y}{2\sqrt{Dt}}\right\}\right\}\sqrt{Dt} \tag{37}$$

In order to integrate Equations (36) and (37) into Eq. (35), using Eq. (30) to replace $K(t,z)$ as $\omega(y,t)$ and according to $\omega = Ce^{\lambda t}$; therefore, we have

$$C(x,t) = Me^{-\lambda t}\left(\frac{e^{-\frac{(x+ut)^2}{4Dt}}}{\sqrt{D\pi t}} - \frac{ue^{\frac{3u^2t+2ux}{4D}}\left(1-erf\left(\frac{x+2ut}{2\sqrt{Dt}}\right)\right)}{D}\right) \tag{38}$$

where $C(x,t)$ is the ^{137}Cs concentration (mBq cm^{-3}), x is the soil depth (cm), t is the time (yr), D is the diffusion coefficient (cm^{-2} yr^{-1}), u is the erosion rate (cm yr^{-1}). We can integrate x and then

$$Arm(T) = Aref(T)\left(erf\left[\frac{u\sqrt{t}}{2\sqrt{D}}\right] - 2e^{\frac{3u^2t}{4D}}\left(erf\left[\frac{u\sqrt{t}}{\sqrt{D}}\right]-1\right)-1\right) \tag{39}$$

where $Arm(T)$ is the remaining ^{137}Cs inventory in the profile (mBq cm^{-2}), and $Aref(T)$ is the ^{137}Cs inventory in the reference profile (mBq cm^{-2}).

5. Model discussion and simulation

5.1. Discussion

If the erosion rate is $u = 0$, Eq. (38) can be rewritten as:

$$C(x,t) = Me^{-\lambda t}\frac{e^{-\frac{x^2}{4Dt}}}{\sqrt{D\pi t}} \tag{40}$$

Eq. (40) has the same solution as the simplified diffusion model under non-corrosive conditions Eq. (5). If the erosion rate $u\neq 0$, then Eq. (38) can be divided into two parts, Eq. (41) and Eq. (42):

$$C1(x,t) = Me^{-\lambda t}\frac{e^{-\frac{(x+ut)^2}{4Dt}}}{\sqrt{D\pi t}} \tag{41}$$

$C1(x,t)$ can be written as $C(x,t)$ in the equation to translate to the $u*t$ units.

$$C2(x,t) = Me^{-\lambda t}\left(\frac{ue^{\frac{3u^2t+2ux}{4D}}\left(1-erf\left(\frac{x+2ut}{2\sqrt{Dt}}\right)\right)}{D}\right) \tag{42}$$

In Eq. (42), to arrange t and u unchanged and to observe $C2(x,t)$ with an increasing changeable trend with depth x; in Fig. 3, $C2(x,t)$ in the figure has the maximum value at the surface, it decrease exponentially with increasing depth.

If one ignores the ^{137}Cs flux variation at the boundary due to itself diffused influence, then $C1(x,t)$ can be interpreted as the distribution function of the ^{137}Cs concentration changes with time and depth under erosion conditions, namely the non-aggressive distribution function (Eq. (40)) makes upward translation of the $u*t$ units. If in the uniform rainfall locations, the erosion function will be longer and it must consider the boundary conditions. $C2(x,t)$ can be seen as an impact factor of the flux variation to the ^{137}Cs concentration under the topsoil, it will decrease exponentially with increasing depth. As noted above, $C1(x,t) - C2(x,t)$ constitutes a complete solution of the ^{137}Cs diffusion equation in abundant rainfall regions and under erosion conditions.

5.2. Model simulation

Eq. (38) and Eq. (39) provide two possible solutions to obtain the erosion rate. Through ^{137}Cs concentration values at different depths, Eq. (38) to provide curve fitting and to directly obtain the erosion rate and the diffusion coefficient. Fig. 4 is a sampling point profile of ^{137}Cs concentration distribution found in; an undisturbed woodland Peak near Xinpu town, Zunyi City, Guizhou Province, China in 2013. The specific radioactivity of the area was 504 (Bq m^{-2}), its background value in 1963 was 2497 (Bq m^{-2}), and its regional background value in 2013 was 802 (Bq m^{-2}). The ^{137}Cs decay constant is taken as 0.023 yr^{-1}, then the data in Fig. 4 and the correlation coefficient are applied to Eq. (38) and Matlab is used to obtain a curve fit, as shown in Fig. 5. This method results in diffusion coefficient of D = 0.54 (cm^{-2} yr^{-1}), an erosion rate of 0.034 (cm yr^{-1}), and a curve fitting index R^2, of 0.92.

When using Eq. (39) to calculate the erosion rate, it is necessary to obtain the diffusion coefficient D through the background profile. To compare the soil erosion rates calculated by the simple transport model and the moving boundary model, we used the 2004 Kaixian data (Zhang et al., 2008), the diffusion coefficient D is taken as 1.98 (cm^{-2} yr^{-1}) and the erosion rate is obtained under a different loss ratio. Table 1 lists the data comparison results.

As previously described, in areas with high rainfall the erosion rates estimated by the simplified transmission model will be greater than the actual values. Under the same ^{137}Cs loss ratio, the results of the moving boundary erosion model in this paper are all less than the former, with values of about 60% of the former. The result is in accord with our expectations.

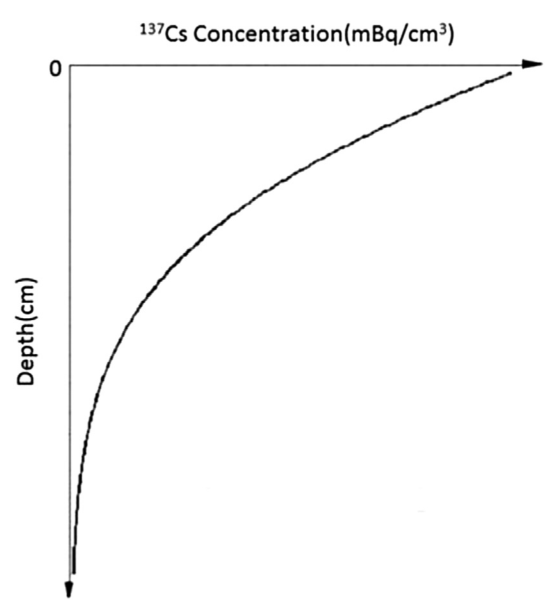

Fig. 3. Concentration changing curve.

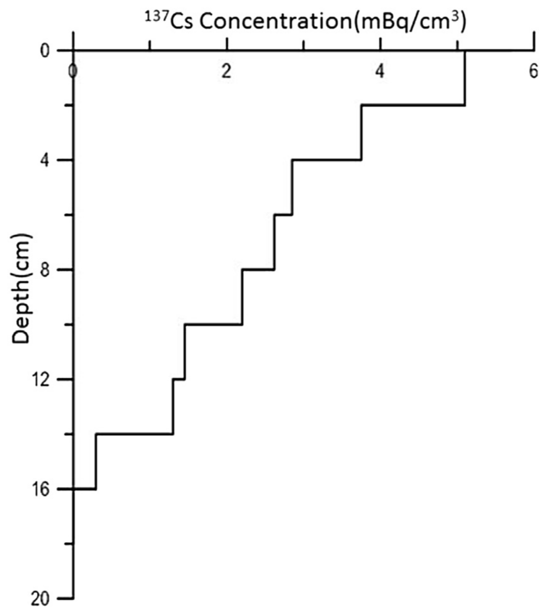

Fig. 4. Sample point concentration distribution.

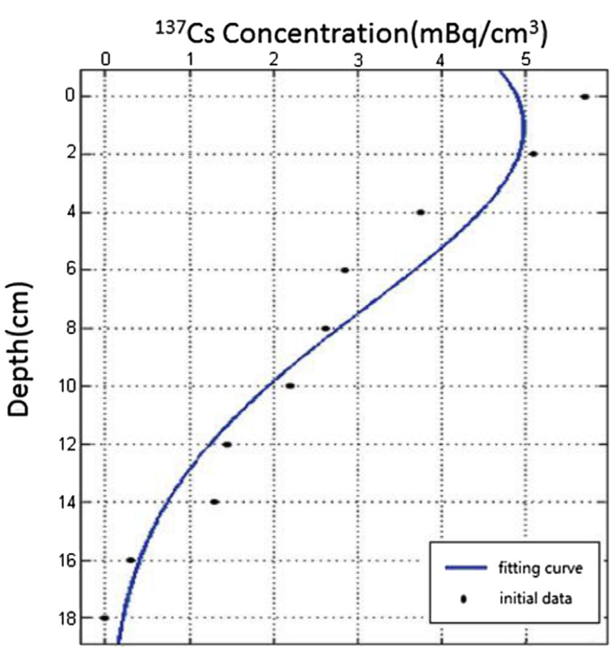

Fig. 5. Model curve fitting.

Table 1
Comparison of the estimated annual soil losses between the simplified transport model, and the moving-boundary model.

^{137}Cs loss proportion (%)	Annual soil loss depth u (cm)	
	A	B
	Simplified transport Model, Eq. (8)	Moving-boundary Model, Eq. (39)
20	0.049	0.029
40	0.111	0.067
60	0.197	0.121
80	0.343	0.215

6. Conclusion

Using the diffused moving boundary principle and analysing ^{137}Cs flux variation at the boundary, a quantitative models between undisturbed soil profile ^{137}Cs concentration distribution and erosion rate was established. The new model provides a better method that uses ^{137}Cs as a tracer material that can be applied in undisturbed land erosion research.

The moving-boundary model shows that, in areas of high rainfall, the erosion process affects the profiles ^{137}Cs diffusion, and it decreases exponentially with the increase of depth. Therefore, this factor must be considered when building erosion models in these areas, otherwise it will lead to inaccurate soil erosion estimates.

The model satisfactorily matches the measured data, and its curve fitting correlation was 0.92. The data show that, in consideration of the erosion effects to ^{137}Cs diffusion, the erosion rate obtained by the moving boundary model is smaller than the rate obtained by the simplified transmission model, which is consistent with our expectations. The model provides two methods, which is curve fitting and analytical solution, in order to calculate the erosion rate and can be flexibly chosen in the actual study. The model can also construct erosion model and provide relevant research ideas for ^{7}Be, ^{210}Pb etc. as a tracer material.

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